# Crystal Structure of $\alpha \mathrm{Zn}_{2} \mathbf{P}_{2} \mathbf{O}_{7}$ 

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#### Abstract

$\alpha \mathrm{Zn}_{2} \mathrm{P}_{2} \mathrm{O}_{7}$ is a hexpartite structure with a unit cell three times as long in the $a$ direction and twice as long in the $c$ direction as that of the parent $\beta \mathrm{Zn}_{2} \mathrm{P}_{2} \mathrm{O}_{7}$. The crystal crystallizes with monoclinic symmetry with unit cell parameters of $a=20.068(15) \AA, b=8.259(6) \AA, c=9.099(8) \AA$ and $\beta=106.35(5)^{\circ}, Z=12$ and the space group is $12 / \mathrm{c}$. A total of 1841 reflections out of a total of 3740 considered had intensities strong enough to be measured. The structure was refined by full matrix least squares in hoth the possible space groups Ic and I2/c. The final reliability index is 0.080 for all the data. The coordination number of the cations to oxygen atoms are either 5 (for 2 Zn ) or 6 (for 1 Zn ) as compared to 6 for all cations in the $\beta$ phase. All the anions shown nonlinear $\mathrm{P}-\mathrm{O}-\mathrm{P}$ bonds whereas in the $\beta$ phase they appear linear because of disorder. The two independent anions have $\mathrm{P}-\mathrm{O}-\mathrm{P}$ angles of $139^{\circ}$ and $148^{\circ}$. The average inner P -O bond length is $1.60 \AA$ while the terminal bond average to $1.52 \AA$. The average error in the individual PO bond lengths is $0.008 \AA$. This structure can be characterized as composed of sequences of layers. A layer like that found in $\alpha \mathrm{Cu}_{2} \mathrm{P}_{2} \mathrm{O}_{7}$ is followed by two layers like those found in $\alpha \mathrm{Mg}_{2} \mathrm{P}_{2} \mathrm{O}_{7}$. The former layer has anions lying on two-fold axes and the cations are coordinated to five oxygen atoms while in the next two layers half of the cations are five coordinate while the anions in these layers show only approximate two-fold axes.


## Introduction

Zinc pyrophosphate, $\mathrm{Zn}_{2} \mathrm{P}_{2} \mathrm{O}_{7}$, is one of a family of divalent metal ion pyrophosphates which are isostructures in their high temperature ( $\beta$ ) phase. The low temperature ( $\alpha$ ) phases fall into three groups each a multipartite, defined by Lefkowitz, Lukaszewicz, and Megaw (1), of its parent phase. These groups consist of $\mathrm{Cu}_{2} \mathrm{P}_{2} \mathrm{O}_{7}$ which is bipartite along $c$ (2), $\mathrm{Mg}_{2} \mathrm{P}_{2} \mathrm{O}_{7}$ (3), $\mathrm{Ni}_{2} \mathrm{P}_{2} \mathrm{O}_{7}$ (4), and $\mathrm{Co}_{2} \mathrm{P}_{2} \mathrm{O}_{7}$, which are bipartite along both $c$ and $a$ and $\mathrm{Zn}_{2} \mathrm{P}_{2} \mathrm{O}_{7}$ which is bipartite along $c$ and tripartite along $a$. The structures of all the $\alpha$ phases except that of $\mathrm{Zn}_{2} \mathrm{P}_{2} \mathrm{O}_{7}$ have been previously determined. The detail structure of the $\beta$ phase of $\mathrm{Co}_{2} \mathrm{P}_{2} \mathrm{O}_{7}$ has not been reported although magnetic measurements are consistent with its being isostructural to the other $\beta$ phases in this series (5). In addition, the structure of $\mathrm{Mn}_{2} \mathrm{P}_{2} \mathrm{O}_{7}(6)$ at room temperature has been refined by Calvo (7) and from NMR measurements on $\mathrm{P}^{31}$ it is known not to have a first order phase transformation between liquid He temperature and room temperature (8).
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The transformation of $\mathrm{Zn}_{2} \mathrm{P}_{2} \mathrm{O}_{7}$ doped with $\mathrm{Mn}^{2+}$ or $\mathrm{Cu}^{2+}$ has been monitored by electron spin resonance (9) and there appears to be a first order transition at $132^{\circ} \mathrm{C}$ and a gradual transformation of the second kind from this temperature to about $155^{\circ} \mathrm{C}$. The detailed atomic changes occurring in this region are uncertain. The transformation of $\mathrm{Mg}_{2} \mathrm{P}_{2} \mathrm{O}_{7}$ features a temperature range in which the high and low temperature phases coexist (10). The diffuse streaks found in the diffraction patterns of $\mathrm{Cu}_{2} \mathrm{P}_{2} \mathrm{O}_{7}$ in the transition region suggests that its transformation may provide the most detailed information about the mechanism (11). In terms of the gross structure $\alpha \mathrm{Zn}_{2} \mathrm{P}_{2} \mathrm{O}_{7}$ is the most complex of the series. An understanding of the phases and transformations of the entire series will be simplified by a study of each member of the series.

## Experiments

## A. Crystal Data

Crystals of $\mathrm{Zn}_{2} \mathrm{P}_{2} \mathrm{O}_{7}$ were prepared from the melt obtained from the decomposition of precipitated $\mathrm{ZnNH}_{4} \mathrm{PO}_{4}$. The crystals were heated in a Vycor
container to about $50^{\circ} \mathrm{C}$ above their melting temperature of $1020^{\circ} \mathrm{C}$ (12) and slowly cooled to about $900^{\circ} \mathrm{C}$. They were then removed from the furnace and allowed to reach ambient temperature. Single crystals were selected for X-ray examination and were observed to show the general extinction conditions $h+k+l$ odd and $l$ odd if $k$ were zero. In addition, in an [100] zone photograph, only a very few $l$ odd could be discerned and these were very weak.

Iattice parameters had been obtained by De Wolff (13) from powder data and reported based upon a $C$ centered cell. We have chosen a body centered cell to preserve the same axial directions for the unit cell as in the $\beta$ phase. Accurate parameters were obtained from a Debye-Sherrer pattern. Crystals were ground into a fine powder, mixed with

TABLE I

## Crystal Data

$$
\begin{aligned}
& a=20.068(15) \AA \\
& b=8.259(6) \AA \\
& c=9.099(8) \AA \\
& \beta=106.35(5)^{\circ} Z=12 \\
& \rho_{\text {exp }}=4.1 \mathrm{gm} / \mathrm{cm}^{3} \\
& \rho_{\mathrm{calc}}=4.2 \mathrm{gm} / \mathrm{cm}^{3} \\
& \mu \text { (linear absorption coefficient) }=110 \mathrm{~cm}^{-1}
\end{aligned}
$$

a solution of acetone and fingernail polish and, when partially dried, drawn into thin fibers. One fiber was used to obtain a 4-day exposure. Sixteen lines were identified using De Wolff's lattice parameters as trial values and the new lattice parameters were obtained by a least squares fit. The lattice parameters are reported in Table I together with other crystal data.

## B. Intensity Data

A crystal of dimensions $0.13 \times 0.13 \times 0.35 \mathrm{~mm}^{3}$ was used to obtain data of the type $h k n, n=0,1,2$, $0 k l$ and $h k k-h / 3$. MoK $\alpha$ radiation was employed together with an integrating precession camera, to record these data. The intensities of all reflections were measured with a Leeds and Northrup G1 microdensitometer and the intensities of symmetry equivalent reflections averaged.
A second crystal with dimensions $0.06 \times 0.06$ $\times 0.11 \mathrm{~mm}^{3}$ mounted parallel to the longest edge was used to record data of the type $h n l, n=0,1,2,3,4,5$ with the Weissenberg camera. MoK $\alpha$ radiation was used to minimize the effects of absorption. Multiple exposures of each layer were taken and the intensities were estimated by visual comparison.

All the data were corrected for the Lorentz and polarization effects and absorption. The absorption corrections were approximate since the crystals were assumed to have idealized geometries. Since the linear absorption coefficient for $\mathrm{MoK} \alpha$ radi-


Fig. 1. The structure of $\alpha \mathrm{Zn}_{2} \mathrm{P}_{2} \mathrm{O}_{7}$ in space group $12 / \mathrm{c}$ is shown projected onto the $a-c$ plane. The layers are labeled $\mathrm{A}, \mathrm{B}$, and C . The oxygen atoms are represented by large open circles and the Zn atoms by small open circles. The filled circles represent phosphorus atom. The bond lengths are shown in $\AA$ units.
ation is $110 \mathrm{~cm}^{-1}$, sizeable errors could be introduced for particular reflections for the former crystals, although the average error was estimated to be of the order to $5 \%$. Standard errors for most of the reflections measured with the densitometer were initially assigned as a constant fraction of the intensity divided by the square root of the number of times it was observed. The standard error of the very weak reflections was increased by a factor accounting for their discrimination from background. The standard errors of the reflections measured visually were also assigned as a constant fraction of the intensity except that the errors for the very weak and the very strong reflections were increased by 50 or $100 \%$ depending upon the estimated accuracy with which the intensity could be observed.

The unit cell of $\alpha \mathrm{Zn}_{2} \mathrm{P}_{2} \mathrm{O}_{7}$ contains 12 formula units and if the space group were Ic there would be three in the asymmetric unit. If the space group were $12 / \mathrm{c}$, one of the anions would necessarily lie on a symmetry element, either a two-fold axis or a center of symmetry, and one pair of the cations would be related by a center of symmetry or would then lie on a two-fold axis, assuming the structural packing is similar to that in the $\beta$ phase. The space group of lower symmetry was chosen for the initial trial in anticipation that if $\mathrm{I} 2 / \mathrm{c}$ were the proper space group it would be recognized in the final atomic coordinates. This was considered to be the cautious approach despite the fact that statistical considerations using of the data of the [010] zone indicated that the structure was centrosymmetric. The

TABLE II
Atomic Parameters for $\alpha \mathrm{Zn}_{2} \mathrm{P}_{2} \mathrm{O}_{7}$
(The estimated standard deviations are enclosed in parentheses.)

|  | $x$ | $y$ | $z$ | $B(T)$ |
| :---: | :---: | :---: | :---: | :---: |
| $\mathrm{Zn}(\mathrm{A})$ | $0.01302(5)$ | 0.31182(22) | -0.02382(11) | - |
| $\mathrm{P}(\mathrm{A})$ | $0.06618(11)$ | 0.02024(43) | 0.20417(23) | - |
| 01(A) | 0 | 0.0886(17) | ${ }^{\frac{1}{4}}$ | 0.88(13) |
| 02(A) | $-0.12407(29)$ | 0.0059(11) | 0.1437(6) | 0.47(7) |
| 03C(A) | $-0.08360(31)$ | $0.1574(11)$ | 0.3928(8) | $0.54(8)$ |
| 03T(A) | $0.05120(37)$ | -0.1393(12) | 0.1197(8) | 0.95(10) |
| $\mathrm{Zn}(1)$ | 0.16348(6) | $0.18296(18)$ | 0.00104(14) | - |
| $\mathrm{Zn}(2)$ | 0.18239(5) | $0.81089(19)$ | -0.02402(12) | - |
| $\mathrm{P}(1)$ | $0.10189(11)$ | $0.50679(45)$ | 0.28423(22) | - |
| $\mathrm{P}(2)$ | $0.23880(10)$ | 0.51842(45) | 0.20192(22) | - |
| 01 | $0.16450(30)$ | 0.5564(12) | 0.2168 (7) | 0.60 (8) |
| 02(1) | 0.04049(29) | 0.4939(11) | 0.1422(6) | 0.31(6) |
| 02(2) | 0.28953(28) | $0.5101(10)$ | 0.3612(6) | 0.51(7) |
| 03C(1) | 0.09400 (33) | $0.6513(11)$ | 0.3829(8) | 0.56(9) |
| 03C(2) | $0.25367(32)$ | $0.6595(11)$ | 0.1090(7) | 0.64(8) |
| 03T(1) | 0.11792(39) | $0.3550(12)$ | 0.3758(9) | 1.11(11) |
| 03T(2) | 0.23510 (37) | $0.3576(12)$ | 0.1162(9) | 0.89(10) |

Anisotropic thermal parameters ( $\AA)^{2 a}$

|  | $U_{11}$ | $U_{22}$ | $U_{33}$ | $U_{12}$ | $U_{13}$ | $U_{23}$ |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| $\mathrm{Zn}(\mathrm{A})$ | $0.0072(4)$ | $0.0052(9)$ | $0.0056(4)$ | $0.0004(5)$ | $-0.0004(3)$ | $0.0004(6)$ |
| $\mathrm{P}(\mathrm{A})$ | $0.0047(7)$ | $0.0049(18)$ | $0.0025(6)$ | $-0.0016(11)$ | $0.0009(8)$ | $0.0003(10)$ |
| $\mathrm{Zn}(1)$ | $0.0056(3)$ | $0.0063(8)$ | $0.0056(3)$ | $0.0009(6)$ | $0.0003(2)$ | $0.0003(6)$ |
| $\mathrm{Zn}(2)$ | $0.0070(4)$ | $0.0038(8)$ | $0.0061(4)$ | $0.0001(6)$ | $-0.0011(3)$ | $0.0001(8)$ |
| $\mathrm{P}(1)$ | $0.0047(7)$ | $0.0053(17)$ | $0.0020(6)$ | $-0.0003(9)$ | $0.0003(5)$ | $0.0010(10)$ |
| $\mathrm{P}(2)$ | $0.0033(6)$ | $0.0072(18)$ | $0.0024(6)$ | $0.0007(8)$ | $0.0007(7)$ | $0.0001(10)$ |

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pseudo-symmetry apparent in the diffraction pattern suggested that this test may not be taken as reliable.

Since the reflections which distinguish the $\alpha \mathrm{Zn}_{2} \mathrm{P}_{2} \mathrm{O}_{7}$ cell from that of the $\beta$ phase were weak, small atomic displacements from the $\beta$ structure should characterize the $\alpha$ phase. It was noted that the anisotropic thermal parameters of the atoms in $\beta \mathrm{Zn}_{2} \mathrm{P}_{2} \mathrm{O}_{7}$ were similar to those in $\beta \mathrm{Cu}_{2} \mathrm{P}_{2} \mathrm{O}_{7}$ (14) and therefore, $\alpha \mathrm{Zn}_{2} \mathrm{P}_{2} \mathrm{O}_{7}$ was regarded tentatively as composed of three layers similar to those in $\alpha \mathrm{Cu}_{2} \mathrm{P}_{2} \mathrm{O}_{7}$. These layers, $1 / 2 a$ (of the $\beta$ cell) thick and infinite in $b$ and $c$, will be designated as layers $\mathrm{A}, \mathrm{B}$, and C. The broadening of the peaks in Patterson projections indicated that the major distortion was along the [103] direction. The cation positions were determined by assuming that in layer $A$ they were displaced from the $\beta$ phase positions as in $\alpha \mathrm{Cu}_{2} \mathrm{P}_{2} \mathrm{O}_{7}$. The remaining cations were split and one half displaced in the positive and the other in the negative [ $10 \overline{3}$ ] direction. The multiplicity of each half cation was varied for one cycle of least squares where the value of

$$
R_{2}\left(=\frac{\sum \omega(|F \mathrm{o}|-|F \mathrm{c}|)^{2}}{\sum \omega|F \mathrm{o}|^{2}}\right)^{1 / 2}
$$

was minimized using a program written for the IBM 7040 by J. S. Stephens. The cation positions whose multiplicity had increased were then assigned to $\mathrm{Zn}^{2+}$ ions. The cations in layer C were located in this manner and those in layer B were found by the use of electron density projections. The cation positions were found to be curiously related to those of the $\beta$ phase. In layer A they had moved by nearly equal and opposite amounts while one cation in each of the remaining layers had essentially remained in its $\beta$ phase position. The other cations in these layers had moved toward each other with the major displacement being $0.3 \AA$ in the ( $10 \overline{3}$ ) direction.

The anions were located by correlating the cationic displacements and that of the central oxygen atom of the anion in the same layers. As has been indicated (2), such a correlation exists because the oxygen atoms shared between the anion and the cation transfer the central oxygen atom displacements into a unique direction of rotation of the cations bonded to this anion. The axis of rotation essentially lies in the plane separating the cations and passes through the phosphorus atom on the near side of the anion. Electron density maps were used to refine these $P$ and oxygen atoms positions. Scattering curves were obtained from the International Tables for X-ray Diffraction (15) and corrected for dispersion using tabulated values (16). No effort was
made to distinguish the intensity of the pair of reflections $h k l$ and $h k l$. Each layer was scaled independently and common reflections were averaged and assigned to the derived groups at random. After all the atoms had been located and satisfied reasonable chemical constraints as to bond lengths and angles, the zinc ions and the phosphorus atoms were assigned anisotropic thermal factor but those of the oxygen atoms remained isotropic. This latter restriction was necessary because of computer size and speed limitations. One least square cycle involving a maximum of 190 variables from a total of 218 parameters and 3470 reflections required 90 min . The weights now were chosen so that $|\mathrm{Fo}-F \mathrm{c}|^{2}$ would be independent of the magnitude of $F$ o. Fractional values of the calculated parameter changes were applied. This was necessary since some of the atoms were highly correlated. Unobserved reflections whose calculated structure factor was less than the minimum observable value were left out of the refinement.

In the refined structure for this space group 8 of the 39 independent atoms had non-positive definite thermal parameters although in all but two cases the offending value was within one or two standard deviations of yielding positive definite tensors. The final $R_{2}$ value of 0.098 was obtained using all the data.

The structure obtained at this point appeared to be composed of an $\alpha \mathrm{Cu}_{2} \mathrm{P}_{2} \mathrm{O}_{7}$-like layer followed by two $\alpha \mathrm{Mg}_{2} \mathrm{P}_{2} \mathrm{O}_{7}$-like layers with the remaining structure generated by the I centering operation as shown in Fig. 1. The two cations in the A layer of $\alpha \mathrm{Zn}_{2} \mathrm{P}_{2} \mathrm{O}_{7}$ were coordinated to five oxygen atoms with the sixth oxygen atom displaced along the axial direction of the supposed octahedron to a distance of 3.2 and $3.4 \AA$ from its assigned cation. In $\alpha \mathrm{Cu}_{2} \mathrm{P}_{2} \mathrm{O}_{7}$ the sixth oxygen atoms lie at $2.9 \AA$ from its cation with the second cation related to the first by a center of symmetry. The other axial cation oxygen bond is $2.36 \AA$ long in the latter compound compared to 2.0 and $2.1 \AA$ found here for the unique axial cation oxygen atom bond to the two cations in layer A. The anion in the $\alpha \mathrm{Cu}_{2} \mathrm{P}_{2} \mathrm{O}_{7}$ structure has a twofold axis. In $\alpha \mathrm{Zn}_{2} \mathrm{P}_{2} \mathrm{O}_{7}$, the anion's geometry in the A layer was highly irregular. The two central $\mathrm{P}-\mathrm{O}$ bonds were 1.65 and $1.54 \AA$ long. The $\mathrm{P}-\mathrm{O}$ 3 C bonds on the two halves of the anion had lengths of 1.59 and $1.49 \AA$ with the longest bond on the same side as the longer $\mathrm{P}-\mathrm{O}(-\mathrm{P})$ bond. The shorter P-O $3 T$ bond, $1.46 \AA$, lay on this half of the anion however as compared to a value of $1.58 \AA$ for this bond on the other half of the anion. Although the estimated standard deviations (e.s.d.'s) at $0.04 \AA$ on

TABLE IV
Bond Distances and Angles in $\alpha \mathrm{Zn}_{2} \mathrm{P}_{2} \mathrm{O}_{7}$
(Estimated standard deviations in parentheses.)

$$
\mathrm{P}_{2} \mathrm{O}_{7}^{4-} \text { group }
$$

| $\mathrm{P}_{2} \mathrm{O}_{7}{ }^{4-}$ group |  |  |  |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| (a) Distances ( $\AA$ ) |  |  |  |  |  |
| " $\alpha$ - $\mathrm{Cu}_{2} \mathrm{P}_{2} \mathrm{O}_{7}$-like" layer |  |  |  |  |  |
| $\mathrm{P}(1)-01$ | 1.603(6) | 01(A)-02(A) | 2.499(7) | 02(A)-03C(A) | 2.515(10) |
| -02(A) | 1.541(6) | -03C(A) | $2.460(8)$ | -03T(A) | 2.537(9) |
| -03C(A) | 1.535(9) | -03T(A) | 2.586(13) | $03 \mathrm{C}(\mathrm{A})-03 \mathrm{~T}(\mathrm{~A})$ | 2.546(13) |
| -03T(A) | 1.512(9) | $\mathbf{P}(\mathrm{A})-\mathrm{P}^{\prime}(\mathrm{A})=$ | 3.000(3) |  |  |
| " $\alpha$ - $\mathrm{Mg}_{2} \mathrm{P}_{2} \mathrm{O}_{7}$-like" layers |  |  |  |  |  |
| $\mathrm{P}(1)-01$ | 1.599(8) | 01-02(1) | 2.443(8) | 02(1)-03C(1) | 2.513(10) |
| -02(1) | 1.518(5) | -03C(1) | 2.471(11) | -03T(1) | 2.528(10) |
| -03C(1) | 1.528(9) | -03T(1) | 2.548(13) | 03C(2)-03T(1) | 2.498(14) |
| -03T(1) | 1.489(10) |  |  |  |  |
| $\mathrm{P}(2)-01$ | 1.566(7) | 01-02(2) | 2.512(8) | 02(2)-03C(2) | 2.525(9) |
| -02(2) | 1.520(5) | -03C(2) | 2.424(11) | -03T(2) | 2.525(10) |
| -03C(2) | 1.518(9) | -03T(2) | 2.504(13) | $03 \mathrm{C}(2)-03 \mathrm{~T}(2)$ | 2.524(14) |
| -O3T(2) | 1.531(10) | $\mathrm{P}(1)-\mathrm{P}(2)=3.046(3)$ |  |  |  |

(b) Angles ( ${ }^{\circ}$ )

| " $\alpha$ - $\mathrm{Cu}_{2} \mathrm{P}_{2} \mathrm{O}_{7}$-like"laycr" |  |  |
| :---: | :---: | :---: |
| 01-P-02 |  | 105.3(3) |
| -03C |  | 103.2(5) |
| -03T |  | 112.2(5) |
| $02-03 \mathrm{C}$ |  | 109.7(4) |
| -03T |  | 112.4(5) |
| 03C-03T |  | 113.3(5) |
|  | $\mathrm{P}-\mathrm{O}-\mathrm{P}$ | $138.7(7)^{\circ}$ |
| " $\alpha$ - $\mathrm{Mg}_{2} \mathrm{P}_{2} \mathrm{O}_{7}$-like layers" |  |  |
|  | Side 1 | Side 2 |
| 01-P-02 | 103.2(4) | 109.0(4) |
| -03C | 104.4(5) | 103.6(4) |
| -03T | 111.1(5) | 107.9(5) |
| $02-03 \mathrm{C}$ | 111.2(4) | 112.5(4) |
| -03T | 114.4(5) | 111.7(4) |
| 03C-03T | 111.8(5) | 111.8(5) |
| $\mathrm{P}-\mathrm{O}-\mathrm{P} 148.4(5)^{\circ}$ |  |  |


| Cation Environment |  |  |  |  |  |
| :--- | :--- | :---: | :---: | :---: | :---: |
| $\mathrm{Zn}(\mathrm{A})-02(\mathrm{I})$ | $2.089(8)$ | $\mathrm{Zn}(1)-02(2)^{d}$ | $2.083(7)$ | $\mathrm{Zn}(2)-02(2)^{d}$ | $2.056(7)$ |
| $-02(\mathrm{I})^{a}$ | $2.060(8)$ | $-02(\mathrm{~A})^{a}$ | $2.051(8)$ | $-02(\mathrm{~A})^{a}$ | $2.032(8)$ |
| $-03 \mathrm{CA}^{c}$ | $2.024(7)$ | $-03 \mathrm{C}(\mathrm{A})^{c}$ | $2.102(7)$ | $-03 \mathrm{C}(2)$ | $2.025(7)$ |
| $-03 \mathrm{TA}^{a}$ | $1.956(8)$ | $-03 \mathrm{C}(1)^{b}$ | $2.032(8)$ | $-03 \mathrm{~T}(2)$ | $2.092(9)$ |
| $-03 \mathrm{C}(\mathrm{I})^{b}$ | $2.060(8)$ | $-03 \mathrm{C}(2)^{e}$ | $2.177(7)$ | $-03 \mathrm{~T}(1)^{b}$ | $1.929(9)$ |
| $-03 \mathrm{~T}(1)^{b}$ | $3.289(9)$ | $-03 \mathrm{~T}(2)$ | $2.097(8)$ | $-03 \mathrm{C}(\mathrm{A})$ | $3.279(9)$ |

${ }^{a}$ Generated by $\bar{x} \bar{y} \bar{z}$.
${ }^{b}$ Generated by $x \bar{y} 1 / 2+z$.
${ }^{c}$ Generated by $\tilde{x} y 1 / 2-z$.
${ }^{d}$ Generated by $1 / 2-x 1 / 2-y 1 / 2-z$.
${ }^{e}$ Generated by $1 / 2-x 1 / 2+y-z$.
these bond lengths are high, the anion does appear to deviate significantly from a two-fold axis. These large e.s.d.'s reflect large correlation coefficients which in some cases were as great as 0.9 .

The two adjacent $\alpha \mathrm{Mg}_{2} \mathrm{P}_{2} \mathrm{O}_{7}$-like layers have cations occurring in pairs sharing two oxygen atoms. One of these cations is coordinated to six oxygen atoms while the second shows five-fold coordination. This dimer group is bonded to a similar group in the same layer through a pair of shared oxygen atoms. In $\alpha \mathrm{Mg}_{2} \mathrm{P}_{2} \mathrm{O}_{7}$, these two dimers are related by a $2_{1}$ axis lying in the sheet of the cations. In $\alpha \mathrm{Zn}_{2} \mathrm{P}_{2} \mathrm{O}_{7}$, the B and C layers do not materially deviate from that found in $\alpha \mathrm{Mg}_{2} \mathrm{P}_{2} \mathrm{O}_{7}$. Thus, if $\alpha \mathrm{Zn}_{2} \mathrm{P}_{2} \mathrm{O}_{7}$ were to have symmetry $\mathrm{I} / \mathrm{c}$ then the cations in layer " A " should be related by a center of symmetry with the anion in the layer lying on the two-fold axis.

The refinement was completed in the higher symmetry space group with the two-fold axis through the anion in the A layer. The final value of $R_{2}$ obtained with weights chosen as $W=(43-1.4$ Fo $\left.+0.015 \mathrm{Fo}^{2}\right)^{-1}$ was 0.089 and all thermal ellipsoids were positive definite. The value of $R(=0.080)$ was found to be a significant improvement over that obtained in space group IC. The bond geometries around the anion had improved in the sense that chemically equivalent bonds were more nearly equal.

A few additional cycles of refinement were carried out varying the anisotropic thermal parameters on the oxygen atoms. However ultimately the thermal ellipsoid became non-positive for 02(2) and 03A and thus this model was rejected.

The final atomic parameters are displayed in Table II while the observed and calculated structure factors for all the data are found in Table III.

## Description of Structure

The gross features of the structures are as discussed for the refinement in the space group of lower symmetry. The bond distances and angles, collected in Table IV, have in fact shifted in a direction such as to yield chemically more reasonable anion geometries. The anion in layer A has a $\mathrm{P}-\mathrm{O}-\mathrm{P}$ bond angle of $139^{\circ}$ with a bridging $\mathrm{P}-\mathrm{O}$ bond of $1.603(6)$ $\AA$ and terminal $\mathrm{P}-\mathrm{O}$ bonds averaging $1.53 \AA$. The bridging $\mathrm{P}-\mathrm{O}$ bonds in the second anion are $1.599(8)$ and $1.566(7) \AA$ with average terminal $\mathrm{P}-\mathrm{O}$ bonds having values of 1.51 and $1.52 \AA$ on side 1 and 2, respectively. Here the $\mathrm{P}-\mathrm{O}-\mathrm{P}$ bond angle is $148^{\circ}$. As has been found in $\alpha \mathrm{Mg}_{2} \mathrm{P}_{2} \mathrm{O}_{7}$ the oxygen atom strongly bonded to only one cation, 03T, show the shortest terminal $\mathrm{P}-\mathrm{O}$ bond length at $1.49(1) \AA$.

Again as in $\alpha \mathrm{Mg}_{2} \mathrm{P}_{2} \mathrm{O}_{7}$ the longest bridging $\mathrm{P}-\mathrm{O}$ bond lies on the same side of the anion as the shortest-terminal $\mathrm{P}-\mathrm{O}$ bond.

The cations showing six-fold coordination have remained in essentially the same place relative to the symmetry elements, as in the $\beta$ phase. The remaining cations have been displaced largely along the direction of the axial bond and as a result, the bond to the oxygen atom in the direction opposite to this displacement is broken. This feature is also found in $\alpha \mathrm{Mg}_{2} \mathrm{P}_{2} \mathrm{O}_{7}$ and $\alpha \mathrm{Cu}_{2} \mathrm{P}_{2} \mathrm{O}_{7}$. The cationic displacements in the $a-c$ plane from the analogous $\beta$ phase positions are $0.27 \AA$ in the $\alpha \mathrm{Cu}_{2} \mathrm{P}_{2} \mathrm{O}_{7}$-like layer and $0.30 \AA$ in the $\alpha \mathrm{Mg}_{2} \mathrm{P}_{2} \mathrm{O}_{7}$-like layer.

## Discussion

It is now apparent that the ordered structures in this series of compounds are intimately related. $\alpha \mathrm{Cu}_{2} \mathrm{P}_{2} \mathrm{O}_{7}$ has a structure where the anion direction is the same for every anion in a sheet. A sheet is the region between planes separated by $1 / 2 c$ (of the $\alpha$ phase) and centered by the cations. In $\alpha \mathrm{Mg}_{2} \mathrm{P}_{2} \mathrm{O}_{7}$ there is an anti-phase boundary after every two layers while in $\alpha \mathrm{Zn}_{2} \mathrm{P}_{2} \mathrm{O}_{7}$ they occur after every third layer. In each case, a unit cell is defined after two such antiphase boundaries. The transformation of $\mathrm{Cu}_{2} \mathrm{P}_{2} \mathrm{O}_{7}$ indicates (11) that phase boundaries occur on the average after eight layers.

The cations in $\alpha \mathrm{Cu}_{2} \mathrm{P}_{2} \mathrm{O}_{7}$ show five-fold coordination with the sixth oxygen atom displaced axially to a distance of $2.9 \AA$. In $\alpha \mathrm{Mg}_{2} \mathrm{P}_{2} \mathrm{O}_{7}$ the symmetry is further reduced with the formation of a sixth additional bond to half the cations. The axial oxygen atom to the remaining cation lies displaced by over $3.3 \AA$. As has been indicated, the $\alpha \mathrm{Zn}_{2} \mathrm{P}_{2} \mathrm{O}_{7}$ shows a mixture of the two structures with sequences defined by $\alpha \mathrm{Cu}_{2} \mathrm{P}_{2} \mathrm{O}_{7}$-like layers separating a pair of $\alpha \mathrm{Mg}_{2} \mathrm{P}_{2} \mathrm{O}_{7}$-like layers. This latter pair are symmetryrelated as in $\alpha \mathrm{Mg}_{2} \mathrm{P}_{2} \mathrm{O}_{7}$ whereas the $\alpha \mathrm{Cu}_{2} \mathrm{P}_{2} \mathrm{O}_{7}$-like layer has the elements of symmetry of the layer in $\alpha \mathrm{Cu}_{2} \mathrm{P}_{2} \mathrm{O}_{7}$. The $\alpha \mathrm{Zn}_{2} \mathrm{P}_{2} \mathrm{O}_{7}$ structure suggests that the "up" "down" sequence of $\alpha \mathrm{Mg}_{2} \mathrm{P}_{2} \mathrm{O}_{7}$-like layers should be regarded as the stabilized arrangement.
A detailed comparison of the anions, shown in Fig. 2, with those in the parent structures indicates a great deal of similarity. The bridging $\mathrm{P}-\mathrm{O}$ bond length has increased from $1.576(7) \AA$ in $\alpha \mathrm{Cu}_{2} \mathrm{P}_{2} \mathrm{O}_{7}$ to $1.603(6) \AA$ in $\alpha \mathrm{Zn}_{2} \mathrm{P}_{2} \mathrm{O}_{7}$. This effect follows from decreased $\pi \rho-\pi d$ bonding predicted by Cruickshank (17) as a result of the $\mathrm{P}-\mathrm{O}-\mathrm{P}$ angle decreasing from $157^{\circ}$, as found in $\alpha \mathrm{Cu}_{2} \mathrm{P}_{2} \mathrm{O}_{7}$, to $139^{\circ}$ as found in $\alpha \mathrm{Zn}_{2} \mathrm{P}_{2} \mathrm{O}_{7}$. The $\mathrm{O}-\mathrm{P}-\mathrm{O}$ bond angles involving the terminal oxygen atoms are all larger than the
$\alpha-\mathrm{Cu}_{2} \mathrm{~F}_{2} \mathrm{O}$,

$\alpha \sim M_{92} \mathrm{P}_{2} \mathrm{O}_{7}$

$\alpha-Z_{N_{2}} \mathrm{P}_{2} \mathrm{O}_{T}$



Fig. 2. The anions found in $\alpha \mathrm{Cu}_{2} \mathrm{P}_{2} \mathrm{O}_{7}$ and $\alpha \mathrm{Mg}_{2} \mathrm{P}_{2} \mathrm{O}_{7}$ are compared with those found in $\alpha \mathrm{Zn}_{2} \mathrm{P}_{7} \mathrm{O}_{7}$. The open circles are oxygen atoms and the filled circles are phosphorus atom. The bond lengths are shown in $\AA$ units.
ideal tetrahedral angle as if the phosphorus atom has been displaced from the center of the tetrahedra to the face formed by the terminal oxygen atoms. In both compounds only the 01-P-03T exceeds the ideal tetrahedral value while the remaining bond angles involving 01 are substantially smaller than $109.5^{\circ}$.

The anion in the $\alpha \mathrm{Mg}_{2} \mathrm{P}_{2} \mathrm{O}_{7}$-like layer shows a $\mathrm{P}-\mathrm{O}-\mathrm{P}$ angle of $148^{\circ}$ as compared to $144^{\circ}$ in $\alpha \mathrm{Mg}_{2} \mathrm{P}_{2} \mathrm{O}_{7}$. Here the bridging $\mathrm{P}-\mathrm{O}$ bond is 1.599(8) $\AA$ on the side showing the shortest $\mathrm{P}-\mathrm{O}$ bond ( $1.489(10) \AA$ ). In $\alpha \mathrm{Mg}_{2} \mathrm{P}_{2} \mathrm{O}_{7}$ these lengths are are $1.612(10)$ and $1.472(10) \AA$. The bridging $\mathrm{P}-\mathrm{O}$ bond on the remaining side is $1.566(7)$ compared to $1.569(10) \AA$ in $\alpha \mathrm{Mg}_{2} \mathrm{P}_{2} \mathrm{O}_{7}$. Both tetrahedra show $\mathrm{O}-\mathrm{P}-\mathrm{O}$ bond angles involving terminal oxygen atom substantially larger than $109.5^{\circ}$. The bond angles involving 01 are quite different on the two sides, with a greater regularity, for those involved with the shorter bridging $\mathrm{P}-\mathrm{O}$ bond.

The phase transformation at $132^{\circ} \mathrm{C}$ generates a $\beta \mathrm{Zn}_{2} \mathrm{P}_{2} \mathrm{O}_{7}$ size cell but with two independent cation sites for the $\mathrm{Mn}^{2+}$ ion. Since the size of the resonance from the two sites seems to have equal amplitudes and the difference in the zero field splitting parameter changes relative to the $\beta$ phase are comparable to those found in $\alpha \mathrm{Mg}_{2} \mathrm{P}_{2} \mathrm{O}_{7}$, it appears that the
structure transforms to one where half of the cations have five-fold coordination and the remainder have six-fold coordination. Since the line widths are very broad in the transformation region there appears to be a variation in the detailed topology about each of these cation sites.

Unfortunately there does not appear to be a structural reason for a change to equal numbers of five- and six-fold coordinated cations. Further the displacement from the high temperature resonance, where all cation sites are six-fold, is equal and opposite for the two sites. The zero field splitting parameters is primarily sensitive to fields produced by the near neighbor environment and thus the five coordinate site is expected to differ much more from the $\beta$ phase than the six coordinate site. Of course we may in fact be seeing directly the effects of the distortions arising from the $\mathrm{Mn}^{2+}$ on the local environment in $\alpha \mathrm{Zn}_{2} \mathrm{P}_{2} \mathrm{O}_{7}$. At any rate the molecular basis of the intermediate phase in $\mathrm{Zn}_{2} \mathrm{P}_{2} \mathrm{O}_{7}$ is not understood.

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[^0]:    ${ }^{a}$ These values were obtained from $\beta_{i j}=2 \pi^{2} b_{i} b_{j} U_{i j}$ where $\beta_{i j}$ 's appear as a temperature effect through exp[ $-\left(\beta_{11} h^{2}+\right.$ $\left.\left.2 \beta_{12} h k+\cdots\right)\right]$.

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